VLBA Test Memo. No. 62

Simulations of Some Types of Holography Errors for VLBA Antennas

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April 30, 1999

Introduction

In VLBA Test Memo 57 (hereafter referred to as Memo1), I presented some options for VLBA antenna surface measurement. As part of Memo1, I included a section describing the different contributions to the measurement error for antenna holography experiments. In this memo, I will present the results of a simulation program which I have created in an attempt to test the theoretical values of the measurement error that I presented in Memo1. In general, I'll investigate the behavior as a function of raster size, wavelength, and the other parameter of interest. The types of errors that I will examine are SNR, pointing, amplitude fluctuation, phase fluctuation, and polarization. I will **not** be examining other types of errors, including receiver errors (other than amplitude fluctuation), bulk reflector shape change errors, source structure related errors, near-field errors, or EM clutter errors (see Memo1). I will also not be investigating subreflector related errors (either rms or offsets), or errors related to the panels themselves (e.g., the effects of panel discontinuities on the determination of offsets, etc...) which are inherent in most implementations of the traditional holography technique (see James *et al.* 1993 and Deguchi *et al.* 1993 for exceptions). Because of this, the simulation I present here is simpler than some which have been used to examine holography in general (see e.g., Rahmat–Samii 1985; Deguchi *et al.* 1993; James *et al.* 1993). I assume throughout that phase is being measured for the holography as well as the amplitude (i.e., no phase–recovery technique is being used).

Description of the Simulation

In order to test how the different sources of error in the measurement of the radiation pattern map themselves into error in the derived surface deviation map, I need a way of producing the radiation pattern, and the surface deviation pattern from that. I produce the radiation pattern by calculating "visibilities" at the sample locations which are determined by the wavelength, raster size, and oversampling factor.

The sample locations (u, v) are given by:

$$u, v = k \ \Delta_{uv} \qquad k = -N/2, -N/2+1, ..., N/2-2, N/2-1 \quad , \tag{1}$$

where N is the raster size. The spacing between samples is:

$$\Delta_{uv} = \frac{\lambda}{\kappa\ D} \quad , \tag{2}$$

where λ is the wavelength, D is the antenna diameter, and κ is the oversampling factor.

The "visibility" can be thought of as being formed between the "test" antenna (the one for which a holographic determination of the surface deviation map is desired) and the "reference" antenna (the one which is always presumed to be on boresight). This complex visibility is comprised of a true signal portion and a noise portion:

$$V(u,v) = V_s(u,v) + n(u,v) \quad . \tag{3}$$

I assume that the signal value on boresight is 1.0, i.e., $V_s(0,0) = 1.0$, and that the noise part does not vary as a function of u and v. The noise visibility is determined by calculating its real and imaginary parts separately:

$$\Re\{n\} = \frac{\xi_1}{\mathrm{SNR}} \qquad ; \qquad \Im\{n\} = \frac{\xi_2}{\mathrm{SNR}} \quad , \tag{4}$$

where SNR is the signal to noise ratio on boresight, and ξ_1 and ξ_2 are gaussian random variables with unit width and zero mean.

The amplitude of the signal portion of the visibility is calculated as:

$$A_s(u,v) = A_r(u,v)\, A_t(u,v)\ [1 + \xi_a\, A_a] \quad , \tag{5}$$

where A_r is the amplitude of the reference antenna radiation pattern, A_t is the amplitude of the test antenna radiation pattern, A_a is the magnitude of the amplitude errors, and ξ_a is a gaussian random variable with unit width and zero mean. The amplitudes of the reference and test antennas are determined by the perturbed values of u and v, where the perturbation occurs from pointing errors. Consider the perturbation on u for the test antenna. For a constant pointing offset (an incorrect global pointing model) of $\Theta_{c_{u,t}}$ and with the magnitude of the rms pointing offset of $\theta_{rms_{u,t}}$, the perturbed value of u_t (the t subscript to indicate the test antenna) is:

$$u_t' = u_t + \Theta_{c_{u,t}} + \xi_p\, \theta_{rms_{u,t}} \quad , \tag{6}$$

where, again, ξ_p is a gaussian random variable with unit width and zero mean. Given a similar perturbation on v_t, the amplitude of the test antenna radiation pattern is then:

$$A_t(u,v) = \frac{J_1(z_t')}{z_t'/2} \quad , \tag{7}$$

where z_t' is given by:

$$z_t'(u,v) = \frac{\pi\, r_t'(u,v)\, D}{\lambda} \quad , \tag{8}$$

with $r'_t(u,v) = \sqrt{u'_t u'_t + v'_t v'_t}$. The amplitude of the radiation pattern for the reference antenna is calculated in a similar fashion, given its pointing offsets and rms values. The phase of the signal portion of the visibility is calculated as:

$$\phi_s(u,v) = \xi_\phi \, \phi_{rms} \quad , \tag{9}$$

where ϕ_{rms} is the magnitude of the phase jitter from point to point between the reference and test antennas, and ξ_ϕ is a gaussian random variable with unit width and zero mean.

Once the radiation pattern has been calculated, the aperture current distribution is obtained by an inverse Fourier transform of that pattern. If an FFT is used, then the data is often oversampled, in order to avoid aliasing (see discussion below). This is where the oversampling factor (κ) introduced above comes in. If a DFT is used, oversampling is not needed, but it is still common to oversample by factors of about 1.2 or so. If a DFT is used in the simulation, the values of the aperture current distribution are calculated at positions on the aperture x and y via:

$$C(x,y) = C(l\Delta_{xy}, m\Delta_{xy}) = \sum_{j=-N/2}^{N/2-1} \sum_{k=-N/2}^{N/2-1} e^{-2\pi ijl/N} \, e^{-2\pi ikm/N} \, V(j\Delta_{uv}, k\Delta_{uv}) \quad , \tag{10}$$

where l and m range from $-N/2$ to $N/2-1$. The spacing between samples is:

$$\Delta_{xy} = \frac{\kappa \, D}{N} \quad . \tag{11}$$

Of course, for locations in the aperture where $\sqrt{x^2+y^2} > D$, the above summation is not done, but the current distribution is simply set to 0. If an FFT is used in the simulation, the only complication is that a shift must be accounted for. This is because available FFT's assume that the input samples go from 0 to $N-1$, rather than $-N/2$ to $N/2-1$. This means that a shift of $N/2$ samples is introduced, which introduces a multiplicative factor which looks like:

$$C'(x,y) = e^{\pi \, i \, (l+m)} C(x,y) \quad , \tag{12}$$

where $C(x,y)$ is the matrix returned by the FFT routine. Effectively, this just multiplies values in the aperture current distribution by $e^{\pi \, i}$ (which is just equal to -1) in a checkerboard pattern over the distribution.

The aperture phase distribution is then the phase portion of the aperture current distribution:

$$\psi(x,y) = \mathrm{Phase}[C(x,y)] = \arctan\left(\frac{\Im\{C(x,y)\}}{\Re\{C(x,y)\}}\right) \quad . \tag{13}$$

In real holography experiments, the aperture phase distribution thus obtained has in it the effects of several types of errors, including subreflector offsets and pointing offsets. I'm not simulating any of the subreflector effects here. Theoretically, the pointing offsets translate exactly into a linear phase gradient across the aperture phase distribution. This can be fit for and subtracted out, and I do just that. So, I fit a linear phase gradient of the form:

$$\psi_l(x,y) = ax + by + c \quad , \tag{14}$$

for the coefficients a, b, and c. I then subtract this from the aperture phase distribution, i.e.,

$$\psi'(x,y) = \psi(x,y) - \psi_l(x,y) = \psi(x,y) - (ax + by + c) \quad . \tag{15}$$

The relationship between the surface errors and the aperture phase is given by (see Memo1):

$$\epsilon(x,y) = \frac{\lambda}{2\,\pi}\,\frac{1}{2\,\cos(\gamma/2)}\,\psi'(x,y) \quad , \tag{16}$$

where γ is the angle formed from feed to subreflector to surface point. For a parabolic reflector with an on–axis feed, this reduces to:

$$\epsilon(x,y) = \frac{\lambda}{4\,\pi}\,\sqrt{1 + \frac{x^2 + y^2}{4\,f^2}}\,\psi'(x,y) \quad , \tag{17}$$

where f is the focal length (8.4 meters for VLBA antennas).

Implementation and Timing

I have implemented this simulation in two ways: as a FORTRAN program, and as an IDL procedure. The FORTRAN program runs much faster (see next paragraph), so is used to investigate the statistics of many simulations of a particular type of error (via multiple calls from a Perl script). The IDL procedure, though slower, gives a nice visual display of the results, and is good for giving a feel for repeatable non–gaussian types of errors.

Table 1 shows the results for the time in seconds taken for the different simulations (done on my Sparc Ultra-1). Where the dashes appear in the table, the times were too fast or too slow to measure reliably. You can see easily that the FORTRAN execution times are *much* faster than the IDL times. This is understandable, since IDL has much more overhead and generality and is an interpreted language (rather than FORTRAN which is a compiled language), and hence almost always runs slower than the equivalent optimized FORTRAN. The slowness of the DFT in comparison to the FFT also comes as no surprise.

Table 1. Time to run the simulations (in seconds)

	FORTRAN		IDL	
N	DFT	FFT	DFT	FFT
16	—	—	2	—
32	1.5	—	30	0.5
64	20	—	500	2.5
128	300	0.5	—	10
256	—	2	—	40

Inputs

The required inputs for the simulations are: the antenna diameter, the wavelength, the raster size, the oversampling factor, the SNR, the phase rms, the amplitude rms, the test and reference antenna pointing offsets in u and v, the test and reference antenna rms values in u and v, the focal length, and the type of transform (DFT or FFT, and whether the data is apodized or not prior to transform - see section below). I presume that the focal length is constant at 8.4 m. All other inputs are allowed to vary (save the transform variables - again, see below).

Outputs

Once the map of surface deviation has been obtained, there are several pieces of information which can be obtained from it. The two I will report here are the rms deviation and maximum deviation across the reflector surface. It is not clear to me which of these is a better indicator of the intrinsic measurement error, which is why I report them both. The reason it is not clear which is better is that there are systematic (non-gaussian) errors introduced in the aperture distribution, and hence simply reporting the rms deviation can be misleading. For instance, given a 128X128 raster, with oversampling by a factor of 1.2 ($\kappa = 1.2$), there are 8945 samples on the aperture. For this many samples, you would expect of order 0.5 samples with a magnitude of 4-σ, i.e., the maximum deviation across the surface should be of order 4-σ. In fact, depending upon the type of error and other input parameters, it can be much higher than this, as will be shown below. For this reason, it seems better to report both pieces of information.

DFT, FFT, oversampling, and apodization

In these simulations, where a fully regular grid of samples is generated for the radiation pattern, there should be no difference between the results for the DFT and the FFT. This is indeed what I find in testing, as shown in Table 2. For this reason, all of the simulations I present here were done with the FFT.

In the holography literature, it is common to run across various statements involving oversampling. It seems that a lore has arisen where oversampling by large factors (like 1.2) is advocated. While the only harm in oversampling is the fact that a penalty is paid in resolution on the antenna surface (resolution in worse by the oversampling factor), and there is actually a reduction in errors (because of the effective smoothing over the surface), in fact, it is not necessary to oversample by such large factors. In well sampled holography maps, it is sufficient to effectively oversample only by 1 sample, i.e., the oversampling factor $\kappa = 1 + 1/N$. This has been recognized by others (see e.g. Morris 1984), but doesn't seem to have propagated through the holography literature and community. As an example, in the VLA holography experiments, Rick Perley uses as oversampling factor of 1.2. Table 2 shows results of simulating the effect of oversampling. This table shows that the improvement in surface deviation error is due almost entirely to the smoothing, i.e., there seems to be little gain from oversampling (at least in the larger N maps). This table also shows simulations where apodization was used for the FFT (also commonly called windowing or smoothing). This is a common technique for suppressing aliasing when dealing with FFT's. The apodization function used was a 2-D Hanning (cosine) function. Apodizing the radiation pattern effectively smooths the aperture phase map, and hence the surface deviation map. For this reason, the errors are reduced in the apodized maps. For Hanning apodization, the smoothing is roughly a factor of 2, so resolution on the antenna surface is about a factor of 2 worse (while errors get better by a bit better than a factor of 2). Better functions could be investigated which would pay less penalty in resolution but still suppress aliasing, but that is beyond the scope of this memo. For the simulations presented in this memo, an oversampling factor of 1.05 was used, with no apodization.

Table 2. Error as a function of transform, oversampling, and apodization.

N	κ	apodized?	DFT/FFT	rms error (μm)	max error (μm)
16	1.0	N	DFT	19.33±1.1	60.50± 8.8
16	1.0	N	FFT	19.38±1.1	60.34± 9.3
16	1.0	Y	FFT	7.48±0.8	24.95± 5.6
16	1.2	N	DFT	13.41±0.8	38.23± 5.8
16	1.2	N	FFT	13.53±0.9	39.50± 6.3
16	1.2	Y	FFT	5.33±0.7	17.67± 4.2
32	1.0	N	DFT	38.38±1.1	138.60± 17.5
32	1.0	N	FFT	38.38±1.1	136.76± 17.6
32	1.0	Y	FFT	14.83±0.7	60.39± 11.1
32	1.2	N	DFT	26.97±0.8	95.25± 12.3
32	1.2	N	FFT	26.91±0.8	95.78± 15.7
32	1.2	Y	FFT	10.47±0.5	41.10± 6.8
64	1.0	N	DFT	77.28±1.1	338.68± 45.3
64	1.0	N	FFT	77.33±0.9	336.33± 52.5
64	1.0	Y	FFT	29.52±0.7	145.08± 22.4
64	1.2	N	DFT	53.91±0.8	225.10± 33.2
64	1.2	N	FFT	53.75±0.8	226.38± 35.4
64	1.2	Y	FFT	20.63±0.6	95.33± 17.4
128	1.0	N	FFT	154.73±1.0	793.45±136.3
128	1.0	Y	FFT	58.30±0.7	313.32± 42.5
128	1.05	N	FFT	140.36±1.0	739.62±128.3
128	1.05	Y	FFT	52.93±0.8	283.31± 42.6
128	1.1	N	FFT	128.30±0.9	708.83±122.8
128	1.1	Y	FFT	48.38±0.7	264.22± 36.5
128	1.2	N	FFT	107.53±0.9	541.80± 79.3
128	1.2	Y	FFT	40.79±0.6	215.51± 29.5

SNR

In Memo1, the theoretical error for a given SNR on boresight was given as:

$$\epsilon_{err} \sim \frac{\lambda\, N}{3\,\pi\, \mathrm{SNR}} \quad .$$

Table 3 shows the errors for the simulations as a function of λ, N, and SNR. For well sampled maps with higher SNR, the proper relationship between the error and other parameters seems to be:

$$\epsilon_{max} \sim \frac{\lambda\, N}{\pi\, \mathrm{SNR}} \quad ,$$

and

$$\epsilon_{rms} \sim \frac{\lambda\, N}{5\,\pi\, \mathrm{SNR}} \quad .$$

Pointing

In Memo1, the theoretical error for a given rms pointing error of magnitude (θ_{rms}) was given as:

$$\epsilon_{err} \sim \frac{\theta_{rms}\, D}{8} \quad . \qquad (18)$$

Table 4 shows the errors for the simulations as a function of λ, N, and θ_{rms} on the antennas. For well sampled maps with relatively good pointing (where the pointing error is roughly better than one tenth of the primary beamwidth, or about $11''$ at 13.4mm, $6''$ at 7mm, and $3''$ at 3.5mm), the proper relationship between the error and other parameters seems to be:

$$\epsilon_{max} \sim \frac{\theta_{rms}\, D}{3} \quad ,$$

and

$$\epsilon_{rms} \sim \frac{\theta_{rms}\, D}{12} \quad .$$

As a test, simulations were also run where constant pointing offsets were present on either or both test and reference antenna. Again, when pointing is good (as defined above), the proper offsets are always recovered in the reduction. When pointing gets bad, however, incorrect constant offsets are derived, and the surface deviation errors go up rapidly. This emphasizes the importance of good pointing for holography.

Amplitude Fluctuations

In Memo1, the theoretical error for a given amplitude fluctuation of magnitude (A_a) was given as:

$$\epsilon_{err} = \frac{A_a\, \lambda}{6\,\pi} \quad . \qquad (19)$$

Table 3. Error as a function of SNR.

λ	N	SNR	rms error (μm)	max error (μm)	theoretical (μm)	max/theoretical	rms/theoretical
13.5	8	50	144.39±15.0	385.32±75.5	229	1.7	0.63
13.5	16	50	303.00±19.9	1112.04±300.2	458	2.4	0.66
13.5	32	50	677.09±24.9	3606.64±313.1	917	3.9	0.74
13.5	64	50	1292.80±19.0	4142.82±65.1	1833	2.3	0.71
13.5	128	50	1731.10±9.7	4194.91±28.9	3667	1.1	0.47
13.5	256	50	1967.37±4.8	4213.17±14.6	7334	0.6	0.27
13.5	8	100	72.15±9.3	185.80±47.5	115	1.6	0.63
13.5	16	100	147.99±9.5	520.97±127.9	229	2.3	0.65
13.5	32	100	299.62±10.1	1532.55±601.4	458	3.3	0.65
13.5	64	100	670.10±14.3	3929.86±174.4	917	4.3	0.73
13.5	128	100	1284.98±10.6	4166.67±28.7	1833	2.3	0.70
13.5	256	100	1730.79±4.4	4204.13±14.5	3667	1.1	0.47
13.5	8	200	35.42±4.1	92.42±17.4	57	1.6	0.62
13.5	16	200	73.61±4.3	252.85±51.4	115	2.2	0.64
13.5	32	200	144.00±4.0	599.54±111.0	229	2.6	0.63
13.5	64	200	293.17±5.0	1850.66±512.6	458	4.0	0.64
13.5	128	200	660.97±6.9	4054.24±84.9	917	4.4	0.72
13.5	256	200	1283.81±4.8	4188.92±17.1	1833	2.3	0.70
13.5	8	400	17.68±2.3	47.13±10.6	29	1.6	0.61
13.5	16	400	36.74±2.4	128.52±29.0	57	2.3	0.64
13.5	32	400	72.17±2.0	301.79±55.5	115	2.6	0.63
13.5	64	400	141.92±2.0	688.41±132.2	229	3.0	0.62
13.5	128	400	287.99±2.4	1991.63±598.3	458	4.3	0.63
13.5	256	400	660.44±3.6	4135.40±45.0	917	4.5	0.72
13.5	128	800	140.51±0.9	726.70±119.0	229	3.2	0.61
13.5	128	1600	69.88±0.5	343.91±43.9	115	3.0	0.61
13.5	128	3200	34.84±0.2	171.89±26.4	57	3.0	0.61
13.5	128	6400	17.43±0.1	84.24±12.1	29	2.9	0.60
7.0	64	50	667.34±10.3	2143.17±36.1	951	2.3	0.70
7.0	128	50	898.27±5.2	2180.36±15.7	1901	1.1	0.47
7.0	64	100	348.25±7.1	2034.11±90.1	475	4.3	0.73
7.0	128	100	667.00±5.4	2165.20±19.5	951	2.3	0.70
7.0	64	200	152.12±2.5	985.16±348.8	238	4.1	0.64
7.0	128	200	343.59±3.5	2100.01±49.4	475	4.4	0.72
7.0	64	400	73.76±0.9	353.07±64.5	119	3.0	0.62
7.0	128	400	149.36±1.2	1064.07±348.8	238	4.5	0.63
7.0	64	800	36.69±0.5	170.66±28.6	59	2.9	0.62
7.0	128	800	72.76±0.5	376.84±60.5	119	3.2	0.61
7.0	64	1600	18.24±0.2	84.51±13.6	30	2.8	0.61
7.0	128	1600	36.17±0.3	184.16±31.1	59	3.1	0.61
7.0	64	3200	9.14±0.1	41.70±6.0	15	2.8	0.61
7.0	128	3200	18.08±0.1	90.29±12.3	30	3.0	0.60
3.5	64	50	333.92±4.6	1071.86±16.4	475	2.3	0.70
3.5	128	50	449.06±2.4	1089.51±8.3	951	1.1	0.47
3.5	64	100	173.83±3.8	1020.28±48.9	238	4.3	0.73
3.5	128	100	333.32±2.2	1081.92±8.1	475	2.3	0.70
3.5	64	200	75.90±1.4	487.93±187.7	119	4.1	0.64
3.5	128	200	171.96±1.8	1055.92±23.4	238	4.4	0.72
3.5	64	400	36.95±0.5	172.92±28.4	59	2.9	0.63
3.5	128	400	74.64±0.5	515.84±164.7	119	4.3	0.63
3.5	64	800	18.35±0.2	84.15±12.5	30	2.8	0.61
3.5	128	800	36.39±0.2	185.75±34.0	59	3.1	0.62
3.5	64	1600	9.15±0.1	43.46±7.4	15	2.9	0.61
3.5	128	1600	18.10±0.1	91.36±12.8	30	3.0	0.60

Table 4. Error as a function of pointing rms error.

λ	N	θ_{rms} (asec)	rms error (μm)	max error (μm)	theoretical (μm)	max/theoretical	rms/theoretical
13.5	8	1	9.65±2.7	21.66±7.2	21	1.0	0.46
13.5	16	1	10.29±2.4	33.01±11.1	21	1.6	0.49
13.5	32	1	9.92±1.7	37.62±11.6	21	1.8	0.47
13.5	64	1	10.22±1.9	42.52±11.2	21	2.0	0.49
13.5	128	1	10.39±2.1	45.86±11.7	21	2.2	0.49
13.5	256	1	10.07±2.2	46.49±12.4	21	2.2	0.48
13.5	8	2	18.36±4.5	41.87±12.5	42	1.0	0.44
13.5	16	2	21.33±4.7	68.50±21.1	42	1.6	0.51
13.5	32	2	21.15±4.6	79.06±24.8	42	1.9	0.50
13.5	64	2	20.30±4.3	85.14±23.9	42	2.0	0.48
13.5	128	2	20.70±4.7	93.44±28.6	42	2.2	0.49
13.5	256	2	20.76±4.4	93.96±22.3	42	2.2	0.49
13.5	8	4	38.54±10.1	86.67±27.4	84	1.0	0.46
13.5	16	4	42.42±10.9	137.18±46.3	84	1.6	0.51
13.5	32	4	39.24±8.6	144.38±43.2	84	1.7	0.47
13.5	64	4	41.54±9.3	177.22±53.7	84	2.1	0.49
13.5	128	4	40.54±8.7	178.56±51.8	84	2.1	0.48
13.5	256	4	39.68±8.0	179.70±47.4	84	2.1	0.47
13.5	64	8	83.91±16.3	374.39±104.5	168	2.2	0.50
13.5	128	8	80.64±17.8	362.95±122.3	168	2.2	0.48
13.5	64	16	174.94±38.5	948.58±566.3	336	2.8	0.52
13.5	128	16	170.11±41.1	882.06±453.1	336	2.6	0.51
7.0	64	1	10.17±1.9	42.45±11.1	21	2.0	0.48
7.0	128	1	10.54±2.4	46.30±13.3	21	2.2	0.50
7.0	64	2	20.75±4.4	88.26±25.1	42	2.1	0.49
7.0	128	2	20.34±3.9	85.73±20.6	42	2.0	0.48
7.0	64	4	41.39±9.1	185.52±46.5	84	2.2	0.49
7.0	128	4	41.47±8.2	183.07±46.3	84	2.2	0.49
7.0	64	8	86.72±20.7	532.11±359.5	168	3.2	0.52
7.0	128	8	82.68±17.9	461.82±264.5	168	2.7	0.49
7.0	64	16	211.12±88.7	1356.08±672.2	336	4.0	0.63
7.0	128	16	213.87±105.3	1412.1± 735.9	336	4.2	0.64
3.5	64	1	10.58±2.6	45.78±14.8	21	2.2	0.50
3.5	128	1	9.75±1.8	43.37±12.1	21	2.1	0.46
3.5	64	2	20.81±4.5	91.16±29.5	42	2.2	0.50
3.5	128	2	19.64±3.7	89.15±24.1	42	2.1	0.47
3.5	64	4	43.26±9.5	221.52±127.9	84	2.6	0.52
3.5	128	4	41.67±10.3	219.79±101.1	84	2.6	0.50
3.5	64	8	110.30±44.5	743.71±360.8	168	4.4	0.66
3.5	128	8	100.55±43.4	662.70±376.4	168	3.9	0.60
3.5	64	16	283.72±127.9	1265.03±330.4	336	3.8	0.84
3.5	128	16	294.76±129.3	1314.50±335.2	336	3.9	0.88

Table 5 shows the errors for the simulations as a function of λ, N, and A_a. For well sampled maps, the proper relationship between the error and other parameters seems to be:

$$\epsilon_{max} = \frac{A_a\,\lambda}{3\,\pi} \quad .$$

and

$$\epsilon_{rms} = \frac{A_a\,\lambda}{15\,\pi} \quad .$$

Phase Fluctuations

In Memo1, the theoretical error for a given phase fluctuation of magnitude (ϕ_{rms}) was given as:

$$\epsilon_{err} = \frac{\phi_{rms}\,\lambda}{6\,\pi} \quad .$$

Table 6 shows the errors for the simulations as a function of λ, N, and ϕ_{rms}. For well sampled maps, the proper relationship between the error and other parameters seems to be:

$$\epsilon_{max} = \frac{\phi_{rms}\,\lambda}{2\,\pi} \quad ,$$

and

$$\epsilon_{rms} = \frac{\phi_{rms}\,\lambda}{12\,\pi} \quad .$$

Polarization Effects

As described in Memo1, observing polarized sources with imperfectly polarized feeds for holography introduces errors in the surface deviation map. Assuming that a source is tracked over a range of parallactic angle that is exactly 90 degrees, and starts perpendicular to the minor polarization ellipse axis, then the effective amplitude rms is about 0.3 (0.289) times the error in ellipticity of the feeds, yielding a theoretical surface error of:

$$\epsilon_{err} = \frac{P_e\,P_s\,\lambda}{10\,\pi} \quad ,$$

where P_e is the ratio of major to minor axis of the circular feed (as a fraction), and P_s is the linear polarization of the source. Assuming that the polarization of the source is 100% (which can be the case when observing celestial maser sources, e.g.), then the errors shown in Table 7 are derived.

Table 5. Error as a function of amplitude fluctuation.

λ	N	A_a (%)	rms error (μm)	max error (μm)	theoretical (μm)	max/theoretical	rms/theoretical
13.5	8	4	10.13±2.4	23.05±6.0	57.3	0.40	0.18
13.5	16	4	11.66±2.7	37.41±12.3	57.3	0.65	0.20
13.5	32	4	11.89±2.6	44.16±14.8	57.3	0.77	0.21
13.5	64	4	12.24±2.1	51.65±13.5	57.3	0.90	0.21
13.5	128	4	11.72±2.0	51.73±13.2	57.3	0.90	0.20
13.5	256	4	11.86±2.2	54.02±11.9	57.3	0.94	0.21
13.5	8	2	5.21±1.3	12.26±3.5	28.6	0.43	0.18
13.5	16	2	5.84±1.2	18.87±6.1	28.6	0.66	0.20
13.5	32	2	5.81±0.9	21.29±5.4	28.6	0.74	0.20
13.5	64	2	5.79±0.9	24.75±6.6	28.6	0.87	0.20
13.5	128	2	6.23±1.1	26.22±6.9	28.6	0.92	0.22
13.5	256	2	6.10±1.2	27.97±7.0	28.6	0.98	0.21
13.5	8	1	2.53±0.6	5.88±1.8	14.3	0.41	0.18
13.5	16	1	2.97±0.7	9.58±3.1	14.3	0.67	0.21
13.5	32	1	2.98±0.5	11.36±3.2	14.3	0.79	0.21
13.5	64	1	3.01±0.5	12.62±3.1	14.3	0.88	0.21
13.5	128	1	2.92±0.5	12.83±2.9	14.3	0.90	0.20
13.5	256	1	3.02±0.5	13.85±2.7	14.3	0.97	0.21
13.5	64	0.5	1.53±0.3	6.42±1.5	7.2	0.89	0.21
13.5	128	0.5	1.44±0.2	6.64±1.7	7.2	0.92	0.20
13.5	64	0.25	0.75±0.1	3.21±0.9	3.6	0.89	0.21
13.5	128	0.25	0.73±0.1	3.26±0.9	3.6	0.91	0.20
7.0	64	4	6.28±1.1	26.95±7.2	29.7	0.91	0.21
7.0	128	4	6.05±1.1	27.54±7.0	29.7	0.93	0.20
7.0	64	2	3.12±0.5	12.77±3.2	14.9	0.86	0.21
7.0	128	2	3.10±0.6	13.87±3.4	14.9	0.93	0.21
7.0	64	1	1.56±0.2	6.56±1.4	7.4	0.89	0.21
7.0	128	1	1.54±0.2	6.96±1.5	7.4	0.94	0.21
7.0	64	0.5	0.80±0.1	3.43±0.8	3.7	0.93	0.22
7.0	128	0.5	0.76±0.1	3.28±0.7	3.7	0.89	0.21
7.0	64	0.25	0.39±0.1	1.64±0.4	1.9	0.86	0.21
7.0	128	0.25	0.39±0.1	1.71±0.4	1.9	0.90	0.21
3.5	64	4	3.18±0.6	13.32±3.7	14.9	0.89	0.21
3.5	128	4	2.99±0.4	13.31±2.9	14.9	0.89	0.20
3.5	64	2	1.56±0.3	6.54±1.9	7.4	0.88	0.21
3.5	128	2	1.55±0.3	6.96±1.7	7.4	0.94	0.21
3.5	64	1	0.77±0.1	3.30±0.9	3.7	0.89	0.21
3.5	128	1	0.77±0.1	3.39±0.8	3.7	0.92	0.21
3.5	64	0.5	0.39±0.1	1.62±0.4	1.9	0.85	0.21
3.5	128	0.5	0.38±0.1	1.67±0.4	1.9	0.88	0.20
3.5	64	0.25	0.19±0.0	0.79±0.2	0.93	0.85	0.20
3.5	128	0.25	0.19±0.0	0.86±0.2	0.93	0.92	0.20

Table 6. Error as a function of phase fluctuation.

λ	N	ϕ_{rms} (deg)	rms error (μm)	max error (μm)	theoretical (μm)	max/theoretical	rms/theoretical
13.5	8	60	395.09±379.4	1076.64±1059.9	750	1.4	0.53
13.5	16	60	422.82±334.1	1615.54±1191.1	750	2.2	0.56
13.5	32	60	383.82±162.3	1901.31±1332.4	750	2.5	0.51
13.5	64	60	367.86±85.6	2086.74±1200.6	750	2.8	0.49
13.5	128	60	367.89±59.1	2156.85±1229.8	750	2.9	0.49
13.5	256	60	407.96±294.8	2496.45±1503.8	750	3.3	0.54
13.5	8	30	174.89±44.7	447.49±121.7	375	1.2	0.47
13.5	16	30	199.83±39.9	757.48±221.7	375	2.0	0.53
13.5	32	30	191.02±36.4	879.29±248.0	375	2.3	0.51
13.5	64	30	193.76±39.8	964.82±251.8	375	2.6	0.52
13.5	128	30	192.33±42.2	939.95±208.9	375	2.5	0.51
13.5	256	30	185.46±34.2	1017.97±264.5	375	2.7	0.49
13.5	8	10	60.10±17.3	142.88±46.6	125	1.1	0.48
13.5	16	10	65.18±15.0	246.62±89.1	125	2.0	0.52
13.5	32	10	68.44±15.5	307.60±113.3	125	2.5	0.55
13.5	64	10	63.57±13.7	327.80±100.2	125	2.6	0.51
13.5	128	10	61.96±14.9	332.53±92.6	125	2.7	0.50
13.5	256	10	62.56±14.3	324.51±93.5	125	2.6	0.50
13.5	64	5	32.15±7.3	159.83±54.2	63	2.5	0.51
13.5	128	5	30.55±8.0	155.92±50.0	63	2.5	0.48
13.5	64	2	12.74±3.3	63.52±22.0	25	2.5	0.51
13.5	128	2	12.93±3.0	64.05±21.5	25	2.6	0.52
13.5	64	1	6.64±1.8	31.79±10.6	13	2.5	0.51
13.5	128	1	6.54±1.8	32.76±11.0	13	2.5	0.50
7.0	64	60	222.88±187.4	1112.92±708.1	389	2.9	0.57
7.0	128	60	213.79±103.8	1345.76±857.1	389	3.5	0.55
7.0	64	30	94.39±16.8	480.26±121.9	194	2.5	0.49
7.0	128	30	97.02±18.3	487.45±119.2	194	2.5	0.50
7.0	64	10	34.24±8.3	177.97±52.8	65	2.7	0.53
7.0	128	10	32.43±7.7	174.32±57.2	65	2.7	0.50
7.0	64	5	17.03±4.5	90.60±30.2	32	2.8	0.53
7.0	128	5	16.61±3.9	85.12±26.7	32	2.7	0.52
7.0	64	2	6.87±1.6	33.66±11.7	13	2.6	0.53
7.0	128	2	6.57±1.6	33.52±10.9	13	2.6	0.51
7.0	64	1	3.35±0.9	16.68±5.2	6.5	2.6	0.52
7.0	128	1	3.35±0.9	16.67±5.6	6.5	2.6	0.52
3.5	64	60	111.60±67.1	585.90±403.9	194	3.0	0.58
3.5	128	60	131.68±129.9	717.19±483.4	194	3.7	0.68
3.5	64	30	49.46±11.1	250.41±62.0	97	2.6	0.51
3.5	128	30	46.43±8.8	251.44±58.4	97	2.6	0.48
3.5	64	10	18.22±4.7	90.23±29.7	32	2.8	0.57
3.5	128	10	17.09±4.6	86.29±26.9	32	2.7	0.53
3.5	64	5	8.75±2.2	45.01±16.4	16	2.8	0.55
3.5	128	5	8.12±2.0	42.71±14.7	16	2.7	0.51
3.5	64	2	3.56±0.8	18.05±5.6	6.5	2.8	0.55
3.5	128	2	3.26±0.8	15.91±5.2	6.5	2.5	0.50
3.5	64	1	1.68±0.4	8.34±3.0	3.2	2.6	0.52
3.5	128	1	1.67±0.4	8.32±2.6	3.2	2.6	0.52

Table 7. Error as a function of feed polarization ellipticity error.

λ	N	P_e (%)	rms error (μm)	max error (μm)	theoretical (μm)	max/theoretical	rms/theoretical
13.5	8	8	4.93 +/- 0.5	11.42 +/- 1.1	34.4	0.33	0.14
13.5	16	8	5.03 +/- 0.5	21.56 +/- 2.2	34.4	0.63	0.15
13.5	32	8	4.11 +/- 0.4	26.59 +/- 2.7	34.4	0.77	0.12
13.5	64	8	3.17 +/- 0.3	28.88 +/- 2.9	34.4	0.84	0.09
13.5	128	8	2.11 +/- 0.2	29.70 +/- 3.0	34.4	0.86	0.06
13.5	256	8	1.49 +/- 0.1	28.69 +/- 2.9	34.4	0.83	0.04
13.5	8	4	2.51 +/- 0.3	5.81 +/- 0.6	17.2	0.34	0.15
13.5	16	4	2.56 +/- 0.3	10.98 +/- 1.1	17.2	0.64	0.15
13.5	32	4	2.09 +/- 0.2	13.55 +/- 1.4	17.2	0.79	0.12
13.5	64	4	1.61 +/- 0.2	14.72 +/- 1.5	17.2	0.86	0.09
13.5	128	4	1.08 +/- 0.1	15.15 +/- 1.5	17.2	0.88	0.06
13.5	256	4	0.76 +/- 0.1	14.65 +/- 1.5	17.2	0.85	0.04
13.5	8	2	1.26 +/- 0.1	2.93 +/- 0.3	8.6	0.34	0.15
13.5	16	2	1.29 +/- 0.1	5.54 +/- 0.6	8.6	0.64	0.15
13.5	32	2	1.06 +/- 0.1	6.84 +/- 0.7	8.6	0.80	0.12
13.5	64	2	0.81 +/- 0.1	7.44 +/- 0.7	8.6	0.87	0.09
13.5	128	2	0.54 +/- 0.1	7.66 +/- 0.8	8.6	0.89	0.06
13.5	256	2	0.38 +/- 0.0	7.43 +/- 0.7	8.6	0.86	0.04
13.5	8	1	0.63 +/- 0.1	1.47 +/- 0.1	4.3	0.34	0.15
13.5	16	1	0.65 +/- 0.1	2.79 +/- 0.3	4.3	0.65	0.15
13.5	32	1	0.53 +/- 0.1	3.44 +/- 0.3	4.3	0.80	0.12
13.5	64	1	0.41 +/- 0.0	3.74 +/- 0.4	4.3	0.87	0.10
13.5	128	1	0.27 +/- 0.0	3.86 +/- 0.4	4.3	0.90	0.06
13.5	256	1	0.19 +/- 0.0	3.75 +/- 0.4	4.3	0.87	0.04
7.0	64	8	1.64 +/- 0.2	14.97 +/- 1.5	17.8	0.84	0.09
7.0	128	8	1.10 +/- 0.1	15.40 +/- 1.5	17.8	0.87	0.06
7.0	64	4	0.84 +/- 0.1	7.63 +/- 0.8	8.9	0.86	0.09
7.0	128	4	0.56 +/- 0.1	7.85 +/- 0.8	8.9	0.88	0.06
7.0	64	2	0.42 +/- 0.0	3.86 +/- 0.4	4.5	0.86	0.09
7.0	128	2	0.28 +/- 0.0	3.97 +/- 0.4	4.5	0.88	0.06
7.0	64	1	0.21 +/- 0.0	1.94 +/- 0.2	2.2	0.88	0.10
7.0	128	1	0.14 +/- 0.0	2.00 +/- 0.2	2.2	0.91	0.06
3.5	64	8	0.82 +/- 0.1	7.49 +/- 0.8	8.9	0.84	0.09
3.5	128	8	0.55 +/- 0.1	7.70 +/- 0.8	8.9	0.87	0.06
3.5	64	4	0.42 +/- 0.0	3.82 +/- 0.4	4.5	0.85	0.09
3.5	128	4	0.28 +/- 0.0	3.93 +/- 0.4	4.5	0.87	0.06
3.5	64	2	0.21 +/- 0.0	1.93 +/- 0.2	2.2	0.88	0.10
3.5	128	2	0.14 +/- 0.0	1.99 +/- 0.2	2.2	0.90	0.06
3.5	64	1	0.11 +/- 0.0	0.97 +/- 0.1	1.1	0.88	0.10
3.5	128	1	0.07 +/- 0.0	1.00 +/- 0.1	1.1	0.91	0.06

For well sampled maps, the proper relationship between the error and other parameters seems to be:

$$\epsilon_{max} = \frac{P_e\, P_s\, \lambda}{12\,\pi} \quad ,$$

and

$$\epsilon_{rms} = \frac{P_e\, P_s\, \lambda}{100\,\pi} \quad .$$

Summary and VLBA numbers

Table 8 shows the derived errors (max and rms) from the simulations. What does this mean in terms of holography on the VLBA antennas? From examination of the above tables, it is clear that of the error terms investigated in this memo, SNR, pointing, and phase fluctuation errors dominate. Let us examine each of these three error terms in turn.

Table 8. Summary of errors in the simulations.

type of error	rms error (μm)	max error (μm)
SNR	$\frac{\lambda\, N}{5\,\pi\,\mathrm{SNR}}$	$\frac{\lambda\, N}{\pi\,\mathrm{SNR}}$
pointing	$\frac{\theta_{rms}\, D}{12}$	$\frac{\theta_{rms}\, D}{3}$
amplitude fluctuation	$\frac{\lambda\, A_a}{15\,\pi}$	$\frac{\lambda\, A_a}{3\,\pi}$
phase fluctuation	$\frac{\lambda\, \phi_{rms}}{12\,\pi}$	$\frac{\lambda\, \phi_{rms}}{2\,\pi}$
polarization	$\frac{\lambda\, P_e\, P_s}{100\,\pi}$	$\frac{\lambda\, P_e\, P_s}{12\,\pi}$

SNR

Assume that maser sources are to be used for the holography, and that there is an H_2O maser available with flux density of 5000 Jy, and an SiO maser available with flux density of 1000 Jy. Also assume that both 64x64 and 128x128 maps should be investigated. Then the errors listed in Table 9 result. This table lists surface deviation errors in μm (max error is in brackets).

Table 9. SNR errors for VLBA holography.

	H_2O maser (13.4mm)	SiO maser (7mm)
$N = 64$	17.5 [85]	70 [350]
$N = 128$	70 [350]	280 [1400]

pointing

Assume that pointing on the VLBA is currently about $8''$, but might be improved to $4''$ with reference pointing in the near future. Then the errors listed in Table 10 result. This table lists surface deviation errors in μm (max error is in brackets).

Table 10. Pointing errors for VLBA holography.

pointing error (asec)	error (μm)
8	80 [320]
4	40 [160]

phase fluctuations

Assume that amplitude fluctuations on the VLBA are 40° (typical) or 20° (exceptional). Also assume again that observations of SiO masers at 7mm and H_2O masers at 13.4mm might be attempted. Then the errors listed in Table 12 result. This table lists surface deviation errors in μm (max error is in brackets).

Table 11. Phase fluctuation errors for VLBA holography.

	H_2O maser (13.4mm)	SiO maser (7mm)
$\phi_{rms} = 40°$	250 [1500]	130 [780]
$\phi_{rms} = 20°$	125 [750]	65 [390]

Conclusion

As concluded in Memo1, it seems evident that it will be very hard to reach the desired accuracy for measurement of the VLBA primary reflector surfaces by using traditional phase-connected holography.

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