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A MODEL OF THE ORION NEBULA DERIVED FROM RADIO OBSERVATIONS

T. K. Menon

Abstract

The 85-foot Howard E. Tatel telescope at the NRAO was used to obtain a detailed intensity distribution across the Orion Nebula at 3.75 cm wavelength. This intensity distribution has been combined with the higher resolution study of Twiss et al. to obtain a model of the Orion Nebula. The values of the total fluxes, at various frequencies, computed from the above model are in satisfactory agreement with observations. An electron density distribution has been derived from the intensity distribution model. These values are compared with those derived by Osterbrock from optical observations and the effect of density fluctuations is discussed.

I. INTRODUCTION

The distribution of matter in the Orion Nebula has recently been studied by Osterbrock and Flather (1959), Dokuchaeva (1959) and by Rishbeth (1958). Osterbrock utilized the ratio of the intensities of the lines λ 3726 and λ 3729 of [O II]. Then, using the most reliable cross-sections for 0+ as given by Seaton and Osterbrock (1957) and assuming an electron temperature of 10,000° K, he derived the projected density. From this projected density, on the assumption of a spherical model with radius 24 minutes of arc, he obtained the radial density distribution. He derived a maximum density of 1.8×10^4 electrons per cm^3 at the center and about 2.6×10^2 per cm^3 at a distance of 24 minutes of arc from the center. When Osterbrock compared these measurements with certain early measurements by Stromgren (1951) of the total intensity of Hβ line at the center of the nebula, he found that the emission measure obtained from his model was larger by a factor of about 7 than Stromgren's value.

Osterbrock, in order to check his model of electron density distribution, then computed the total flux from the whole nebula at six different frequencies and compared the computed values with the available measurements at the same frequencies. Here again he found that his model predicted fluxes that were, at various frequencies, about 5 to 30 times the observed fluxes. Osterbrock's interpretation of this result is discussed in a later section.

Dokuchaeva (1959) measured the energy density of the Hα radiation from the Orion Nebula using photographs of the nebula, and from this derived electron densities of 1.3×10^3, 4.7×10^2 and 3.7×10^2 electrons per cm^3 for three concentric regions with

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dimensions 56, 260 and 430 square minutes of arc respectively. A discussion of these measurements and the comparsion with radio observations will be taken up in a later paper.

The detailed measurement of the brightness distribution at optical wavelengths is made difficult because of the irregular absorption across the nebula. Radio measurements provide a way out of this difficulty. However, in the region of radio frequencies, until recently, the available angular resolution of radio telescopes was not high enough for any detailed studies. The availability of a 3.75 cm receiver along with the 85-ft. Howard E. Tatel telescope at the NRAO changed the situation to some extent. The half-power beamwidth of the telescope at 3.75 cm is 6 minutes of arc. This means that we can get significant details by taking observations 3 minutes of arc apart. Since the maximum radius of the nebula is of the order of 30 minutes of arc we can expect to obtain a fair approximation to the true intensity contours.

II. OBSERVATIONS

The present study is based on measurements made with the 85-foot telescope during the spring and summer of 1959. The receiver used was a traveling-wave tube radiometer of the same type as that described by Ewen and Drake (1958). The observations consisted of drift curves in right ascension at fixed declinations and scans in declination at fixed right ascensions. Preliminary observations indicated that the declination of the center of the nebula was approximately -5° 26!6. Drift curves at the sideral rate were taken every 3 minutes in declination at seven declinations spaced symmetrically on both sides of -5° 26!6. In right ascension the angular distance covered was usually about 2°. At least 4 drift curves were taken at each declination. Corrections had to be applied to the indicated antenna positions. These corrections, which depended on hour angle, had been determined empirically from observations of a number of radio sources by the NRAO staff. The declination scans were used to check the self-consistency of the coordinate corrections for the Orion Nebula. The overall accuracy of the corrected positions is estimated to be ± 1 minute. The intensity calibration was done by injecting a 1° K signal from a noise source at the front end of the receiver.

In order to convert the observed antenna temperature into intensities or brightness temperature we have to know the beam efficiency of the antenna. The beam efficiency. E_b is defined as

$$E_b = \int_{\substack{\text{main}\\\text{beam}}} f d\Omega \Big/ \int_{4\pi} f d\Omega$$

where f is the radiation pattern of the antenna. A direct determination of E_b has not yet been possible for the 85-ft. reflector at Green Bank. However, we can get a fair idea of its value by the following precedure. The numerator in equation (1) can be evaluated from measurement of the main beam pattern using a point source or by using analytical approximations to the beam shape. The denominator can be estimated by comparing observations of fluxes of sources for which reasonably accurate values are available in the literature.

A Bessel function approximation with halfwidths of 6 minutes of arc was used for the beam pattern and the numerator was found to be 1.372×10^{-2} square degree. The equation for the antenna temperature of a source of flux S, mks units, can be written as

$$T_a = \frac{\lambda^2 E_b S}{2k \int_{\substack{main \\ beam}} f d\Omega} \quad . \tag{2}$$

The flux of Cas A at 3.75 cm is not too well known at present. Observations by Haddock et al. (1957) indicated a value of about 4×10^{-24} mks units while Razin and Plechkov (1957) gave a value of 4.6×10^{-24} mks units. Both the above values fall below the spectral curve with an index of -0.8 suggested by Whitefield (1957). At present the value of 4.6×10^{-24} seems to be better than the value of 4×10^{-24} if the spectral curve is correct. The higher value has been adopted for the purposes of this investigation. Substituting all the known parameters in equation (2) we then get

$$T_a \text{ (Cas A)} = 56.1 \, E_b \quad . \tag{3}$$

The observed antenna temperature for the Cas A source for the 85-ft. telescope was 19°. Hence

$$E_b = 0.34 \quad .$$

The first source of error is in the assumption of the Bessel approximation for the antenna pattern. Recent measurements of the antenna pattern by C. M. Wade (1960) suggest an accuracy of about 5% for the Bessel representation of the pattern. The second major error is in the adopted value of the flux for the Cas A source. Here the estimation of the probable error is difficult due to lack of details of the procedure used by Razin and Plechkov. However, judging from the spectral curve, a probable error of ±10% seems not unreasonable. The third source of error is in the measurement of antenna temperature for Cas A for the 85-ft. telescope. It is believed that the calibration of this temperature is better than ±5%. But any systematic error in the measurement of T_a does not affect the final calibration of intensities of the Orion Nebula, since this systematic factor cancels out in the final determination of intensities or brightness temperatures.

In Table 1 are given the measured antenna temperatures at various positions. The estimated probable error of a single value is ±0.2° K. Using the efficiency of 34% we can convert the antenna temperature into apparent brightness temperatures. An improved approximation to the true distribution can be obtained from Table 1 after correcting for the antenna pattern. The method of correction used in the present paper is that described by Bracewell (1955) for two dimensional distributions. Figure (1) is the contour diagram obtained after correction for antenna pattern.

The most striking feature of the contour diagram is the high degree of symmetry up to a radius of about 6 minutes of arc. The filter photographs by Wurm and Rosino (1959) certainly indicate that the brightest part of the nebula is quite symmetrical, with the absorption towards the northeast part of the nebula being mostly in the foreground. Their photograph number 6 in the continuum at λ 5200 shows the essential continuity of the main emission parts of the nebula beneath the absorption. Although the contours of the outer parts of the nebula are quite distorted some correlation with several optical features is obvious. The absorption northwest of the Trapezium coincides with a con-

TABLE 1
Observed Antenna Temperatures for the Orion Nebula
(Units °K)

α \ δ	-5°35'.6	-5°32'.6	-5°29'.6	-5°26'.6	-5°23'.6	-5°20'.6	-5°17'.6
$5^h28^m\ 0^s$							
12							
24					0		
36					0.1		
48					0.1		
29 0					0.2		
12					0.2		
24		0			0.2		0
36		0.2			0.3		0.1
48		0.3		0	0.6		0.2
30 0		0.3	0	0.1	0.6		0.4
12		0.2	0.1	0.1	0.4	0	0.3
24		0.2	0.3	0.3	0.3	0.1	0
36	0	0.2	0.4	0.5	0.4	0.1	0
48	0.2	0.2	0.4	0.5	0.4	0.3	0.1
31 0	0.5	0.3	0.4	0.5	0.4	0.5	0.4
12	0.3	0.2	0.4	0.3	0.3	0.4	0.4
24	0.2	0.1	0.3	0.4	0.4	0.3	0.2
36	0.1	0.1	0.3	0.4	0.5	0.4	0.2
48	0.3	0.2	0.4	0.4	0.4	0.4	0.2
32 0	0.5	0.2	0.5	0.7	0.5	0.5	0.1
12	0.5	0.3	0.5	0.9	0.6	0.6	0.3
24	0.4	0.5	0.4	0.8	0.6	0.8	0.4
36	0.4	0.7	0.9	1.1	1.2	1.1	0.4
48	0.7	0.7	2.3	2.1	2.6	1.2	0.6
33 0	1.0	0.7	3.6	5.3	5.4	1.6	0.9
12	1.1	0.8	5.4	10.0	8.5	2.4	1.2
24	0.8	1.3	5.8	10.0	7.6	3.3	1.1
36	0.6	1.6	4.0	6.0	4.5	3.2	0.7
48	0.5	0.9	2.1	2.2	2.0	1.0	0.4
34 0	0.4	0.4	0.9	0.4	1.0	0.7	0.2
12	0.1	0.3	0.5	0.4	0.6	0.5	0.4
24	0.5	0.3	0.4	0.5	0.4	0.3	0.5
36	0.8	0.2	0.2	0.8	0.1	0.2	0.5
48	0.4	0.2	0.1	0.4	0.1	0.2	0.5
35 0	0	0.2	0.1	0.5	0.1	0.3	0.5
12		0.1	0.1	0.4	0.2	0.2	0.3
24		0.1	0.2	0.4	0.4	0.2	0.3
36		0.2	0.2	0.3	0.4	0.4	0.2
48		0.2	0.1	0.4	0.4	0.4	0.1
36 0		0.1	0	0.3	0.4	0.3	0.4
12		0.1		0.2	0.2	0.3	0.4
24		0		0.1	0.3	0.5	0.2
36				0.1	0.4	0.3	0
48				0.2	0.3	0.2	
37 0				0.0	0.2	0.2	
12				0.1	0.1	0.1	
24				0.1	0	0.1	
36				0		0	
48							
38 0							

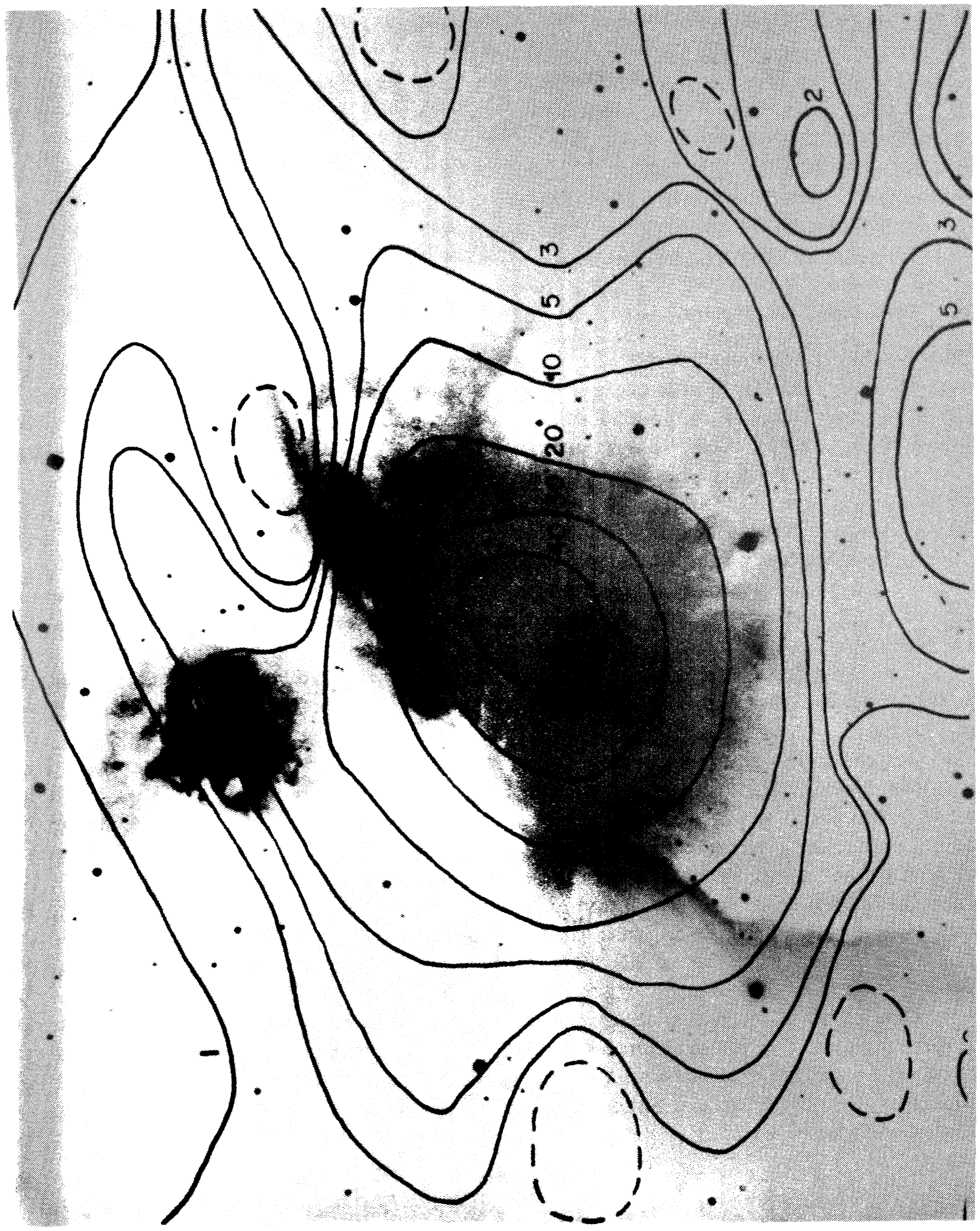

Fig. 1. 3.75 cm brightness temperature contour diagram of the Orion Nebula after correction for antenna smoothing (units °K). The direct photograph is No. 6 of the filter photographs by Wurm and Rosino (1959).

striction of the radio contours whereas the similar absorption east of the trapezium seems to have no effect on the contours. This would seem to indicate that the absorbing material northwest of the Trapezium enters into the nebula and decreases the amount of ionized material there. It also suggests that the arc of absorption seen in the photographs is inclined to the line of sight with the upper end actually entering the nebula.

The center of the contours and the position of Θ' Orionis are given below:

	α (1959)	δ (1959)
Center of 3.75 cm contours	$5^h\ 33^m\ 17^s \pm 4^s$	$-5°\ 26' \pm 1'$
Position of Θ' Orionis	$5^h\ 33^m\ 15^s.5$	$-5°\ 24'.78$

The two positions coincide within the accuracy of the present observations. The high degree of symmetry of the central regions of the nebula suggested the possibility of deriving a spherically symmetrical model for the nebula. However, an angular resolution of 6 minutes of arc is not quite sufficient to derive the true brightness distribution. Recently Twiss, Carter and Little (1960) have published an interferometer study of the brightness distribution in the East-West direction of the Orion Nebula at 1427 Mc/s with an effective angular resolution of about 1 minute of arc. Their results show "that the source has a simple, approximately Gaussian, shape with a width of 3 minutes of arc between the points at half-intensity." There is some asymmetry, with the eastern edge having higher intensity than the western edge. This is indicated by the 3.75 cm contours also. At 1427 Mc/s most of the radiation appears to come from a region less than 12 minutes of arc across. The higher sensitivity of our measurements at 3.75 cm shows that there is a definite faint extension of the nebula beyond a radius of 6 minutes.

A mean distribution of intensity as a function of distance from the center was first obtained from the 3.75 cm contour diagram alone. The intensity scale was adjusted so that the flux computed from the mean distribution was the same as that obtained by actual integration of the contours. Next the shape of the distribution for the region up to 6 minutes of arc from the center was changed to that obtained by Twiss et al. Again the intensity scale was adjusted to give the same flux as the observed flux. By this empirical procedure a mean distribution up to 10 minutes of arc was obtained. The distribution form 10' to 22' was obtained from different considerations discussed below. The final distribution is shown in Fig. 2.

The mean distribution of Fig. 2 is the starting point for all further interpretation in this paper. The absorption coefficient at radio frequencies for free-free transitions in an ionized gas has been calculated by Smerd and Westfold (1949) and more recently by Oster (1959) and Scheuer (1960). The general formula for the absorption coefficient can be written as

$$K_\nu = \left(\frac{4 n_i\, n_e\, Z_i^2 e^6}{3[2\pi\,(mkT_e)^3]^{1/2}\,.\,c\nu^2} \right) \ln A \qquad (4)$$

where n_i and n_e denote the number of ions and electrons per cm^3, Z_i the mean charge of the ion, m denotes the mean reduced mass, T_e the electron temperature, ν is the frequency and other symbols have their usual meanings. The logarithmic term has been defined in different ways by various authors. According to Smerd and Westfold

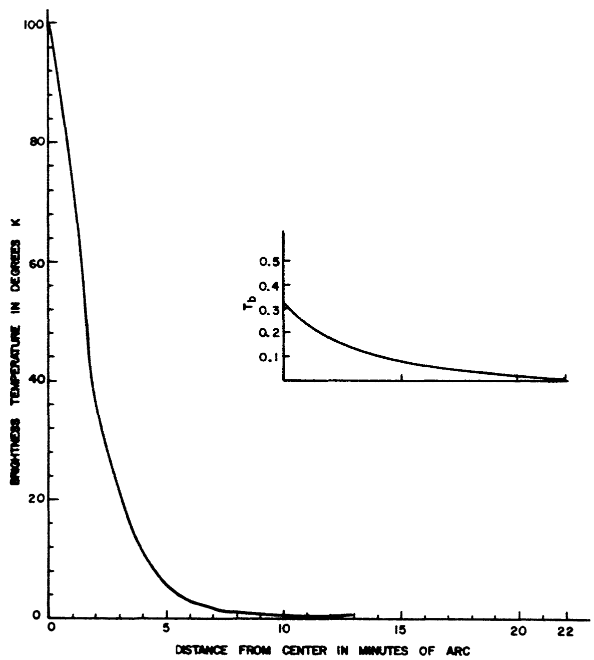

Fig. 2. Mean intensity distribution model for the Orion Nebula at 3.75 cm.

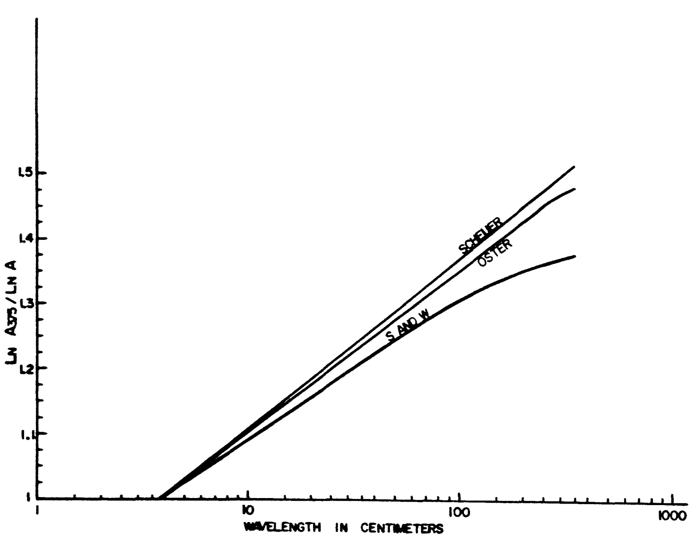

Fig. 3. Ratio of ln $A_{3.75}$ to ln A as a function of wavelength computed according to the theories of Smerd and Westfold, Oster, and Scheuer.

$$\ln A = \ln(1 + v_{01}^2) \qquad (5)$$

where

$$v_{01} = \frac{4kT_e d}{e^2} \quad \text{for } D < 1 \qquad (6)$$

$$v_{01} = \frac{4kT_e d}{e^2} \; \frac{15^{1/2}}{32} \; \frac{1}{D} \quad \text{for } D > 1 \qquad (7)$$

$$d = n_e^{-1/3} \quad \text{and } D = \nu d \; \frac{m}{2kT_e}^{1/2} . \qquad (8)$$

According to Oster

$$\ln A = \left[\ln .425 \; \frac{(kT_e)^{3/2}}{\nu e^2 \sqrt{m}} \right] \qquad (9)$$

and according to Scheuer

$$\ln A = \ln \left(\frac{8k^3 T_e^3}{\pi^2 e^4 \nu^2 m} \right) -2.885 . \qquad (10)$$

Values of ln A were computed using equations (5), (9) and (10) for a value of 10,000° for T_e and given in Table 2 for seven different frequencies. In equation (5) a value of 1000 for n_e was used. It is seen that there is a difference of up to 9% in the values of ln A computed according to the various formulations. However, in our computa-

TABLE 2

Values of ln A

Frequency in Mc/s	ln A Smerd Westfold	$\frac{\ln A}{\ln A_{3.75}}$	ln A Oster	$\frac{\ln A}{\ln A_{3.75}}$	ln A Scheuer	$\frac{\ln A}{\ln A_{3.75}}$
8000	21.3637	1	18.8478	1.0	17.7861	1
3190	23.2035	1.086	20.6866	1.098	19.6252	1.103
1390	24.8651	1.164	22.3482	1.186	21.2865	1.197
960	25.6054	1.199	23.0882	1.225	22.0268	1.238
600	26.5451	1.243	24.0280	1.275	22.9667	1.291
400	27.3563	1.280	24.8390	1.318	23.7777	1.337
85	29.3770	1.375	27.9370	1.482	26.8789	1.511

tions below we are interested only in the ratios of the values of ln A at different frequencies to the value at 3.75 cm. These ratios are plotted in Fig. 3 which shows that the flattest spectrum will be that obtained by using the formulation of Smerd and Westfold. Since the formulations given by Oster and Scheuer differ very little, it was decided to use Scheuer's values for ln A in the present paper.

The brightness temperature at a distance r from the center of the nebula where the optical depth is τ (r) is

$$T_b(r) = T_e(1 - e^{-\tau(r)}) \quad . \tag{11}$$

The above expression assumes that the electron temperature is constant along the enfire line of sight. This is not nevessarily true and we shall see later the consequences of a radial variation of electron temperature. At the wavelength of 3.75 cm the optical depth at the center of the nebula is very small and hence we can use the following approximation for equation (11).

$$\tau(r) = \frac{T_b(r)}{T_e} \quad . \tag{12}$$

By definition

$$\tau = \int k_\nu \, ds \quad , \tag{13}$$

where the intergral is over the pathlength through the nebula. From equations (4) and (10) and assuming $n_i = n_e = n$ we obtain

$$\int n^2 ds = E(r) = 240 \, T_e^{1/2} \, T_b(r) \quad , \tag{14}$$

where E denotes emission measure as defined by Stromgren. According to Stromgren (1951) the emission measure over a circle of 0!65 radius at the center of the nebula has the mean value of 8×10^6. From the present observations the average value of E over a similar area is approximately 2×10^6. The discrepancy between these two values is probably due to the presence of a small shapr peak at the center of the intensity distribution which may not be indicated even with an angular resolution of 1 minute of arc.

The ratio of the optical depths at two frequencies ν_1 and ν_2 is, from (13) and (4),

$$\frac{\tau_2(r)}{\tau_1(r)} = \frac{\nu_1^2 \ \ln A_2}{\nu_2^2 \ \ln A_1} \quad . \tag{15}$$

Hence, if we know the optical depth at any frequency, we can compute the optical depth at any other frequency. From equation (12) we can compute the optical depth at 3.75 cm for any value of electron temperature. Then from (15) and (11) we can calculate the brightness temperature T_b as a function of radius at any frequency we desire. The total flux is given by the expression

$$S = \frac{2k\nu^2}{c^2} \int T_b \, d\Omega \qquad (16)$$

$$= \frac{2k\nu^2}{c^2} \int_o^r T_b(r) \, 2\pi r dr. \qquad (17)$$

The available observational values, which are of sufficient accuracy, of the total flux of the Orion Nebula are given in Table 3. Only those published values for which complete details of the measurement precedure are available are included. Without these details it is impossible to estimate the reliability of the published values. This is particularly so in those cases where the beamwidths of the antennas used were comparable to the source size and hence substantial corrections have to be applied to the directly measured values.

TABLE 3

Observed and Computed Flux densities of Orion Nebula

λ (cm)	$S\text{-}wm^{-2}(c/s)^{-1}$	
	Observed	Computed
3.75	3.93×10^{-24}	3.93×10^{-24}
10.2	4.0×10^{-24}	4.34×10^{-24}
22	4.2×10^{-24}	4.43×10^{-24}
31.25	3.6×10^{-24}	4.30×10^{-24}
75	2.3×10^{-24}	3.07×10^{-24}
350	0.74×10^{-24}	0.72×10^{-24}

The observational data are taken from the following sources:

3.75 cm: NRAO (1959)
10.2 cm: Sloanaker and Nickols (1960)
22 cm: Hagen, McClain and Hepburn (1954)
31.25 cm: Bolton (1960)
75 cm: Seeger, Westerhout and van de Hulst (1954)
350 cm: Mills, Little and Sheridan (1956) as modified by Pawsey (1957)

In deriving a final model of the brightness temperature distribution for the Orion Nebula the parts of the distribution beyond 10 minutes of arc could not be established accurately due to limitations of receiver sensitivity. At the centimeter wavelengths the optical depth even at the center of the nebula is so small that the faint extension contributes little to the total flux. However, at the long wavelengths contributions from the fainter parts are significant, due to increased optical thickness. Hence it was decided to adjust the intensity and extent of the fainter parts by comparing the computed and observed fluxes at the long wavelengths. This adjustment procedure was greatly facilitated by evaluation of equation (17) using the Bendix G-15D digital computer at the NRAO.

Table 4 gives the optical depths as a function of radius for six wavelengths for an electron temperature of 10,000° K. Table 3 gives the computed and observed fluxes. Fig. 4 is a plot of the flux against wavelength. The solid curve is that computed from the present model for an electron temperature of $10^{4°}$ K and the crosses are the observational points. The accuracy of the observational points is about ±20% in all cases. The agreement with observations is quite good at the centimeter wavelengths below 50 cm, but there is not as good agreement at longer wavelengths. It does not seem possible that better agreement can be obtained at long wavelengths by adjustment of the fainter parts alone. First of all the assumption of circular symmetry for the fainter parts may not be justified and departure from this assumed symmetry will result in a lower flux for the observed value than the value computed on the basis of symmetry. Secondly we have assumed a uniform electron temperature through the nebula and this is unlikely to be correct. There is already some evidence for a variation in temperature (Pronyk 1957) within the nebula. Pronyk's results seem to indicate a decrease of electron temperature from a value of 11,000° K near the Trapezium to a value of 7500° K at a distance of 11 minutes of arc from the cluster and a possible increase to a value of 8000° K at a distance of 16 minutes. Aller and Liller (1960) suggest that a value of 9000° K for the electron temperature is preferable. Recent observations at

TABLE 4

Optical Depths τ (r) of Orion Nebula Model for $T_e=10^{4°}$

	3.75 Cm	10.2 Cm	22 Cm	31.25 Cm	75 Cm	350 Cm
0'	.009924	.082002	.408839	.859690	5.30726	130.619
1	.007410	.061229	.305270	.637030	3.96280	97.5300
2	.003560	.029416	.146660	.306050	1.90390	46.8570
3	.002084	.017220	.085854	.179160	1.11450	27.4300
4	.001069	.008833	.044040	.091901	0.57169	14.0700
5	.000587	.004850	.024183	.050464	0.31392	7.72610
6	.000288	.002380	.011865	.024760	0.15402	3.79070
7	.000192	.001587	.007910	.016506	0.10268	2.52710
8	.000128	.001058	.005273	.011004	0.06845	1.68470
9	.000059	.000488	.002431	.005072	0.03155	0.77656
10	.000032	.000264	.001318	.002751	0.01711	0.42118
11	.000023	.000190	.000948	.001977	0.01230	0.30273
12	.000017	.000140	.000700	.001462	0.00909	0.22375
13	.000015	.000124	.000618	.001290	0.00802	0.19743
14	.000011	.000091	.000453	.000946	0.00588	0.14478
15	.000007	.000058	.000288	.000602	0.00374	0.09213
16	.000006	.000050	.000247	.000516	0.00321	0.07897
17	.000004	.000033	.000165	.000344	0.00214	0.05265
18	.000003	.000025	.000124	.000258	0.00164	0.03949
19	.000002	.000017	.000082	.000172	0.00107	0.02632
20	.000002	.000017	.000082	.000172	0.00107	0.02632
21	.000001	.000008	.000041	.000086	0.00053	0.01316
22	.000000	.000000	.000000	.000000	0.00000	0.00000

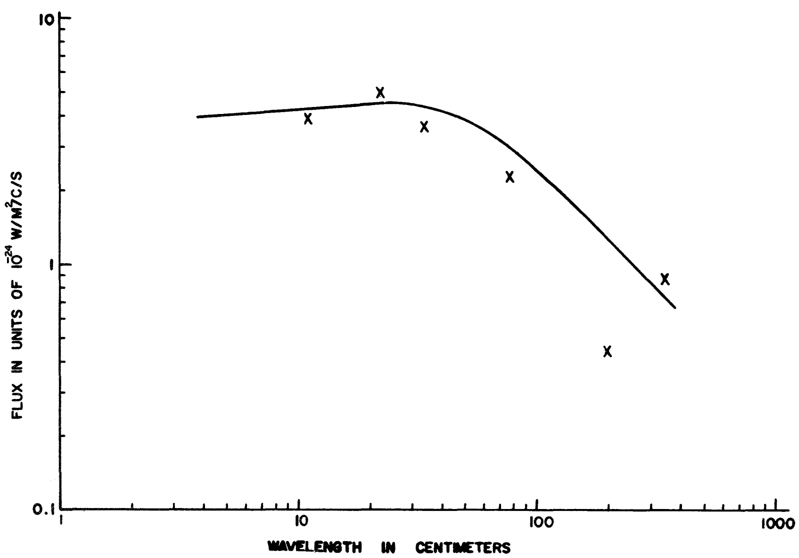

Fig. 4. Observed (crosses) and computed (line) flux densities for the Orion Nebula. Computed fluxes are for the model of Fig. 2 for a value of T_e = 10,000°.

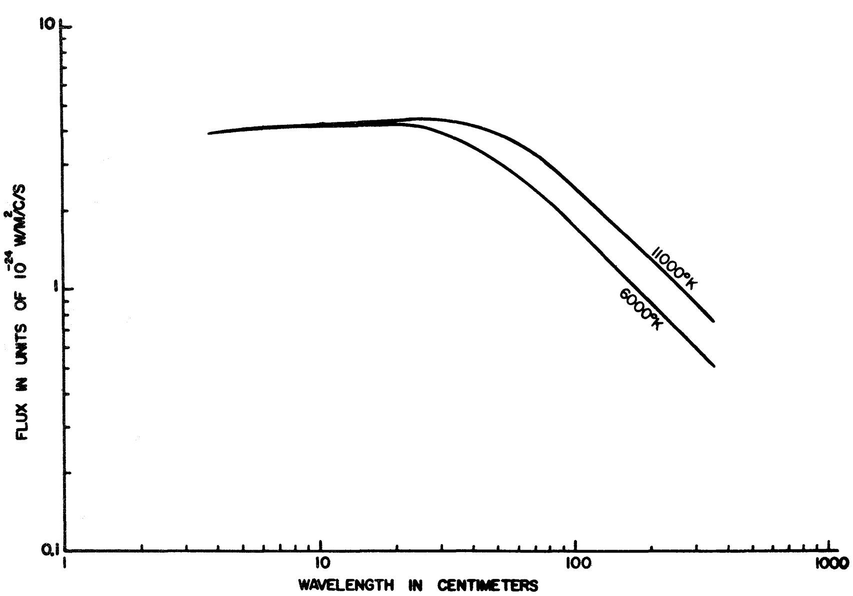

Fig. 5. Computed flux densities for the model of Fig. 2 for values of T_e = 11,000° and 6,000°.

85 Mc by Mills, Little and Sheridan (1956) also seem to indicate electron temperatures as low as 6000° for some H II regions. From equation (15) it is seen that a variation in temperature through the nebula will not significantly change the values of the flux at the shorter wavelengths, but at wavelengths where the nebula becomes optically thick the temperature variation will have large effects. In order to estimate this, fluxes were computed from the model of Fig. 2 for electron temperatures from 11,000° to 6000° K in steps of 500°. Fig. 5 shows the curves for 11,000° and 6,000° K and it is seen that the maximum variation occurs in the region greater than 50 cm wavelength. The optical depth reaches a value of 1 at about 50 cm wavelength for an electron temperature of 10,000° K. An inspection of Fig. 5 shows that it is possible to obtain perfect agreement with observations if we assume a small variation in electron temperature through the nebula. However, the observational values are not sufficiently accurate to warrant such a model at the present time.

We can use the brightness temperature distribution of Fig. 2 to derive the electron density distribution through the nebula on the basis of certain simplifying assumptions. For this purpose the nebula was assumed to be spherically symmetrical and was divided into 22 concentric shells of equal angular width. The distance to the nebula was taken to be 450 parsecs and hence 1 minute of arc = 0.131 pc. The emission measure distribution obtained from equation (14) for $T_e = 10^{4°}$ K was then used to derive the electron densities in successive shells. The actual computations were performed on a digital computer. Fig. 6 shows the derived electron density distribution. The maximum density at the center is about 2.3×10^3 and the density at the distance of 21.5 minutes of arc is found to be about 10 electrons per cm^3.

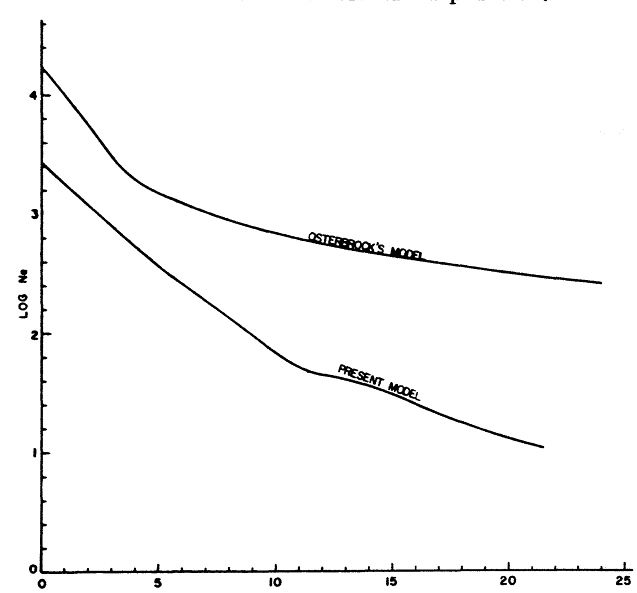

Fig. 6. Distribution of electron density log N_e derived from model of Fig. 2 compared with the distribution of log N_e derived by Osterbrock from optical observations.

The electron density distribution obtained by Osterbrock (1959) is also plotted in

Fig. 6. Osterbrock had interpreted the discrepancy between the computed fluxes for his density model and the observations as being due to the existence of density fluctuations. He then assumed a model with condensations distributed at random through the nebula. These condensations were assumed to have the densities derived from the forbidden line ratios. The effect of this assumption is to decrease the optical depth compared to the uniform model. Osterbrock's model is defined by a parameter α which is the fraction of the line of sight occupied by the condensations. He found that a value of 1/30.3 for α gave the best fit with the observations at centimeter wavelengths. The observational values of the flux at the short wavelengths which he used are too high when compared with more recent measurements. This is because he applied corrections to the published values, most of which had already been partially corrected for source sizes and antenna patterns. It is believed that the values quoted in Table 3 have substantially higher accuracy than the earlier values. If we use these new values, a smaller value of α will have to be used in order to reconcile Osterbrock's model with the observed values at the centimeter wavelengths. Even then the discrepancy at long wavelengths cannot be reconciled. Osterbrock had suggested that fluxes at long wavelengths computed from his model can be decreased further by reducing the area of the nebula in the model. A number of attempts were made to reduce the area while using a single value of α for the whole nebula in order to improve the fit of the predicted fluxes from Osterbrock's model with the observed values. These attempts were not successful.

A possible interpretation of this difficulty is suggested by comparing the electron density distributions shown in Fig. 6. As pointed out before, one may assume that the electron densities obtained from optical studies are the values averaged over a small number of condensations and that the radio studies give the root mean square densities in the line of sight through each shell. The areas over which the averaging has been done are of quite different orders of magnitude in the two cases. The ratio of these two values at any specific point in the nebula is an indication of the degree of fluctuations and the separation of the fluctuations. It should be borne in mind that the actual condensations can have a still higher value than given by Osterbrock. By comparing the brightness distributions given by Osterbrock's model and the present model, we can get some idea of the volume occupied by condensations at various distances from the center. It is seen that the square of the ratio of the electron densities obtained from the present model and the densities obtained by Osterbrock defines a parameter which is essentially equivalent to the parameter α defined by Osterbrock. If R is the radius of a condensation and D is the mean distance from a condensation to its nearest neighbor, then (see Chandrasekhar, 1943)

$$\frac{R}{D} = \frac{\alpha^{1/3}}{0.893} \tag{18}$$

Fig. 6 shows that the value of R/D reached a maximum value of 0.47 at a distance of 3.75 minutes of arc from the center. Filter photographs by Wurm and Rosino (1959) show clearly the fluctuations in density in the central parts of the nebula. Wurm and Rosino remark that outside the central region the density is more homogenous. The outer parts again show extreme filamentary structure. There is therefore qualitative support for suggesting that the parameter introduced by Osterbrock is unlikely to be a constant throughout the nebula. Wurm and Rosino's filter photographs, particularly Nos. 1, 2 and 4, show clearly that the character of the fluctuation is quite different in the central regions and in the outer regions. The fluctuations at the center have a mottled appearance with the largest fluctuations having sizes of the order of 20 seconds of arc or 0.043 parsec. Before we can draw any further conclusions from the above discussion a detailed analysis of the optical data will be needed. Also, the analysis of the velocity data of Wilson, Munch, Flather and Coffeen (1959), which is being done by von Hoener (1960) can provide invaluable information about the statistical description of the gas motions inside the Orion Nebula.

The mass of the Orion Nebula computed from the present model is about 100 $M_\odot$; the hydrogen to helium ratio found by Mathis (1957) was used to obtain the mass per free electron. As pointed out by Osterbrock and others earlier, the gaseous component of the Orion Nebula appears to be less than the stellar content of the system. We have as yet no full information about the stellar content (see Strand, 1958; Parenago, 1953) but a minimum value of about 1000 solar masses is indicated. There does not appear to be any reason to doubt that the gaseous component of the Trapezium cluster is the remnant of the original cloud from which the cluster was formed. As pointed out by Wilson et al., the question then arises as to whether one can derive the initial density distribution which evolved into the present distribution by simple expansion. It is difficult to answer this question in the absence of a detailed theory of the dynamics of H II regions. It might be possible to derive the initial density distribution from a study of the stellar density distribution in the cluster. This aspect will be discussed in a subsequent paper.

We know of very few H II regions with central emission measures of the order of 10^6. There is every reason to believe that the initial density distribution, at the time of formation of an O star inside a contracting interstellar cloud, will have a maximum at the center of the cloud and that the numerical values of the densities may not vary by any large factor in different cases. Hence the paucity of high density H II regions is probably due to the short lifetime of such regions. This had been implied by Shajn (1955) in his considerations of the stability of H II regions. The age of the Trapezium cluster has been variously estimated to be from 10^4 years to 3×10^5 years by Parenago (1953) and by Strand (1958). In the early stages of the evolution of an H II region gravitational forces may have some influence. Shajn has suggested that internal magnetic fields play a prominent role in the Orion Nebula. Parenago and Strand have apparently found evidence for a possible rotation of the cluster. It is likely that the gas, at least in the central parts, takes part in this rotation. The filaments of the outer parts of the nebula have predominantly radial structure for a considerable distance and then they break up into loops at the outermost emission boundary in most long exposure photographs. The data of Wilson, et al., indicate that the Orion Nebula is expanding with a velocity of about 10 km/sec. There is also a suggestion of a small decrease in expansion velocity in the outer parts of the nebula. It seems reasonable to assume that the radial filaments are an indication of expansion and that there is probably also a constraining force, such as can be attributed to a magnetic field, operating on the filaments. It is clear therefore that a complete theory of the evolution of the Orion Nebula faces enormous difficulties.

Observationally we can hope to study the detailed structure of a variety of H II regions using optical and radio methods. We can also approach empirically the problem of the evolution of H II regions by computing the radio frequency spectrum of H II regions with different density distributions. The results of such computations, together with comparsions with Stromgren's theory of H II regions, will be reported in a subsequent paper. Ultimately such studies can provide us with information on the processes of condensation in the interstellar medium and subsequent dispersal of part of these condensations back into the general interstellar medium.

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